SOLUTION OF PROBLEMS OF UNSTEADY HEAT CONDUCTION ON ELECTRICAL MODELS

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A method is described of calculating the nonlinearity of boundary conditions of the third kind in solving problems of unsteady heat conduction on electrical models with RC networks.

At present, many technical problems of unsteady heat conduction are solved by methods of analog simulation on R and RC networks [1–3]. In both cases a finite difference approximation on the left side of the unsteady heat conduction equation is carried out (second spatial derivatives), while the first derivatives with respect to time are solved in various ways which, in the final analysis, affects the means of obtaining a solution. In the one case the solution is obtained continuously on RC networks, and in the second case, on R networks, as a result of successive calculation.

In computational practice RC networks are more widely used, due to there being less difficulty in obtaining a solution. The usual means of performing the calculations is by commercially available network models of special analog computers and integrators of original design, e.g., the SEI-01 of the TsKTI (Central Boiler-Turbine Institute) [3].

In solving the main class of heat conduction problems, e.g., in determining the temperatures of the components of steam and gas turbines, boundary conditions of the third kind are usually employed, i.e., the temperatures T_m of the medium washing the body and the heat transfer coefficients, α, are assigned, since these quantities more suitably reflect conditions of convective heat transfer. The boundary conditions of the third kind vary with time, i.e., $\alpha = f(t)$ and $T_c = f_1(t)$. It is known, also, that an unsteady heat conduction problem is nonlinear, since the physical constants appearing in the equations describing the phenomenon are functions of temperature:

$$\frac{\partial}{\partial x}\left(\lambda\frac{\partial T}{\partial x}\right)+\frac{\partial}{\partial y}\left(\lambda\frac{\partial T}{\partial y}\right)+\frac{\partial}{\partial z}\left(\lambda\frac{\partial T}{\partial z}\right)=c\gamma\frac{\partial T}{\partial t}, \quad (1)$$

$$\alpha\left[T_c(t)-T_n(t)\right]=-\lambda\frac{\partial T}{\partial n}. \quad (2)$$

Unfortunately, in modeling with RC networks, the nature and arrangement of the equipment does not allow variation of the physical constants and of one of the constituent boundary conditions of α with time. In practice, in the solution of many technical problems, λ, c, and γ are assumed to be constant. However, in determining the temperature fields of turbine components using heat transfer coefficients constant with time, considerable error may arise in certain cases.

Work performed by the authors on the electrical integrator of the analog simulation laboratory of Kiev State University has permitted the development of a method of successive approximations for allowing for variation of the heat transfer coefficients in solving unsteady heat conduction problems on RC networks.

The essence of the method is that, in solving unsteady heat conduction problems with boundary conditions of the third kind varying with time, one component (α) remains constant, while the second (T_m) is variable, the law $T_m = f(t)$ being corrected from the condition of conservation with time of the true heat fluxes at the boundaries of the body. We divide the time interval, in which the unsteady process being examined takes place, into several intervals. For the i-th interval we write down the values of the heat fluxes at the boundaries of the body. In solving problems with constant initial values α_0 and variable values of heat transfer coefficients α_1,

$$q_i^1=\alpha_0(T_{m_i}-T_{0_i})F, \quad (3)$$

$$q_i=\alpha_i F(T_{m_i}-T_i). \quad (4)$$

All the quantities appearing in (3) are known. In (4) the wall temperature is unknown, but an approximate value of q_i may be written, using the wall temperature T_{0i}, obtained in a calculation with α_0 = const. Having an approximate value of q_i, we can correct the value of the heat flux in solving with α_0 = const in the i-th time interval, using in the calculation the fictitious temperature of the medium T_m^1, determined from the equality

$$q_i\simeq\alpha_i F(T_{m_i}-T_{0_i})=\alpha_0 F(T_{m_i}^1-T_{0_i}), \quad (5)$$

$$T_{m_i}^1=\frac{\alpha_i}{\alpha_0}(T_{m_i}-T_{0_i})+T_{0_i}. \quad (6)$$

We need to make the same conversion for all the remaining time intervals. By assigning in the solution a new temperature curve converted in this way, we obtain, for points lying on the surface, a new dependence of variation of temperatures $T_1 = f(t)$, corresponding to the second approximation effected. By substituting T_1 into (6) in place of T_{0i}, we obtain a converted medium temperature for the third approximation, and so on. The above process of approximation is convergent, as indicated by repetition of values of converted medium temperatures for two successive approximations.

Original Data of Problems of Cooling and Heating of an Infinite Cylinder

Quantity	Dimensions	Heating	Cooling
Diameter of infinite cylinder	m	0.35	0.35
λ	W/m .degree	31.8	31.8
$a = \lambda / c\gamma$	m^2/sec	$2.1 \cdot 10^{-2}$	$2.1 \cdot 10^{-2}$
Duration of the process	sec	900	900
Range of variation of α	$W/m^2 \cdot$ degree	106–530	530–106
Law of variation of α	—	$\alpha_i = 106 + 0.472$	$\alpha_i = 530 - 0.472$

The method described for successive approximations for solution of problems on RC networks, with allowance for variation of heat transfer coefficients, is applied as follows:

1. The usual preliminary calculation of the unsteady process with α_0 = const is performed.
2. Temperatures at all points on the boundaries of the region are measured.
3. A conversion according to (6) of the temperature curves is carried out for all the boundary points.
4. New temperature curves $T^1_m = f(t)$ are assigned in calculating the next approximation, instead of $T_m = f(t)$.
5. Temperatures in the second approximation at all the surface points are measured. Conversion of the temperature curves of the medium is performed, based on the values obtained, and so on.

In principle the solution may be carried through with any constant boundary resistances, but in this case we must put values of α corresponding to the assumed boundary resistances into (6) in place of the values α_0. It should be noted that, to obtain an accurate solution by the method of successive approximations, we must have the functional transform for assignment of $T^1_m = f(t)$ at each boundary point of the model.

The method of successive approximations has been used to solve problems in the cooling and heating of an infinite cylinder. For comparison, we also solved the problems by the Liebmann method, i.e., by direct calculation of the variation of the heat transfer coefficients. The original data on these problems are shown in the table.

Figure 1 shows graphically the results of comparison of the above-mentioned calculations, results being given for all the approximations effected. The first approximation for the calculations of heating and cooling corresponds to solutions which have been obtained up till now, in using constant heat transfer coefficients, appropriate to steady conditions. The largest discrepancy in the first approximation with regard to results of calculation of similar problems by the Liebmann method constituted 9.2% in heating, and 30% in cooling. The values of the discrepancies were determined with respect to the maximum temperature in the cylinder in the steady regime. In solving the cooling problem in the second approximation, the maximum divergence was 14.5%, in the third, 5.85, in the fourth, 2, and in the fifth, <1%. The discrepancy obtained in the fifth approximation, expressed in degrees, did not exceed 1–2°, which practically falls in the range of accuracy of solution of an unsteady heat conduction problem by the Liebmann method. In calculating heating, the maximum deviation in the second approximation was 1.7%, while in the third approximation the calculated temperatures agreed to an accuracy corresponding to that of solution by the Liebmann method. The slower convergence in cooling is due to the fact that in this case the discrepancies obtained in the first approximation are considerably greater than the discrepancies in the heating calculations. We may accelerate convergence of the solution in cooling by use of the mean integral heat transfer coefficients $\alpha = \frac{1}{t}\int_0^t \alpha(t)\,dt$ in the calculation, the initial conditions, i.e., the steady temperature field, being calculated preliminarily with α_0 corresponding to the condition mentioned. In this case even the third approximation, as in heating, gives results differing from those being compared by not more than 1–2°.

Using the ideas which underlie the method of successive approximations described, we may somewhat facilitate solution of problems of unsteady heat conduction by the Liebmann method, for regions with a developed system of boundary conditions of the third kind. The Liebmann method allows us to take account of variation with time of the physical constants of the material and of the heat transfer coefficients. However, in solving technical problems with allowance for only variation of α, for complex regions, appreciable difficulties arise in connection with the need to vary a large number of boundary resistances in each time interval, which takes up a good deal of time, and may be the cause of accidental errors. The conversion formula for determining the boundary temperature will have, in this case, for the i-th interval of solution, the following form:

$$T^1_{m_i} \approx \frac{\alpha_i}{\alpha_0}\left(T_{m_i} - T_{m_{i-1}}\right) - T_{i-1}. \qquad (7)$$

Expression (7) is not exact, since in it we use the temperature of a boundary point corresponding to i – – 1, i.e., to the preceding time interval. In practice the solution is accomplished in the following order:

1) A model of the region under examination is chosen and initial conditions are assigned.

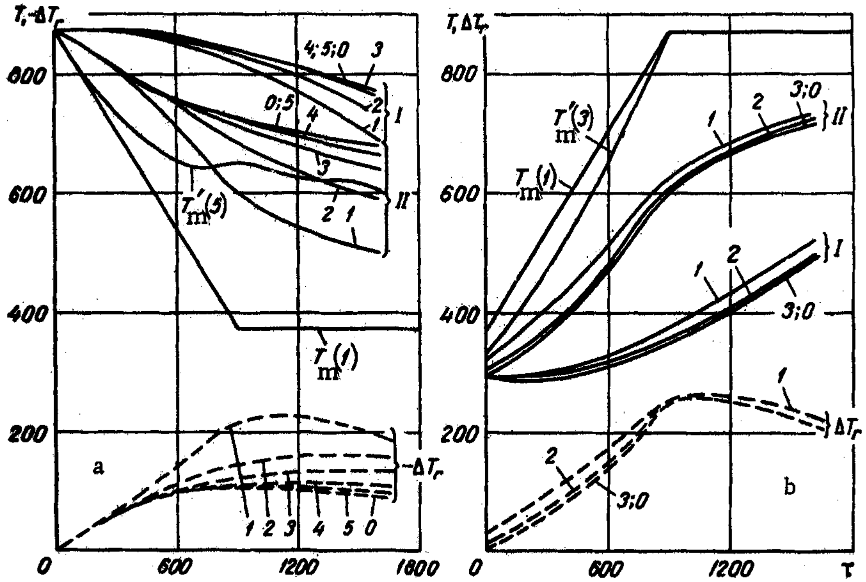

Fig. 1. Results of calculation of cooling (a) and heating (b) of an infinite cylinder: 0—by the Liebmann method; 1, 2, 3, 4, 5—respectively, by the 1- 5th approximations according to the method described; I—temperature on cylinder axis; II– temperature of external surface of cylinder; ΔT_r—temperature drop along cylinder radius from surface to axis; $T_m{}^{(1)}$—variation of temperature of cooling (a) and heating (b) media according to the conditions of the problem; $T'_m{}^{(5)}$ and $T'_m{}^{(3)}$—variation of temperature of medium, calculated by the method described, respectively for cooling in the fifth and for heating in the third approximations: T in ° K, τ in sec.

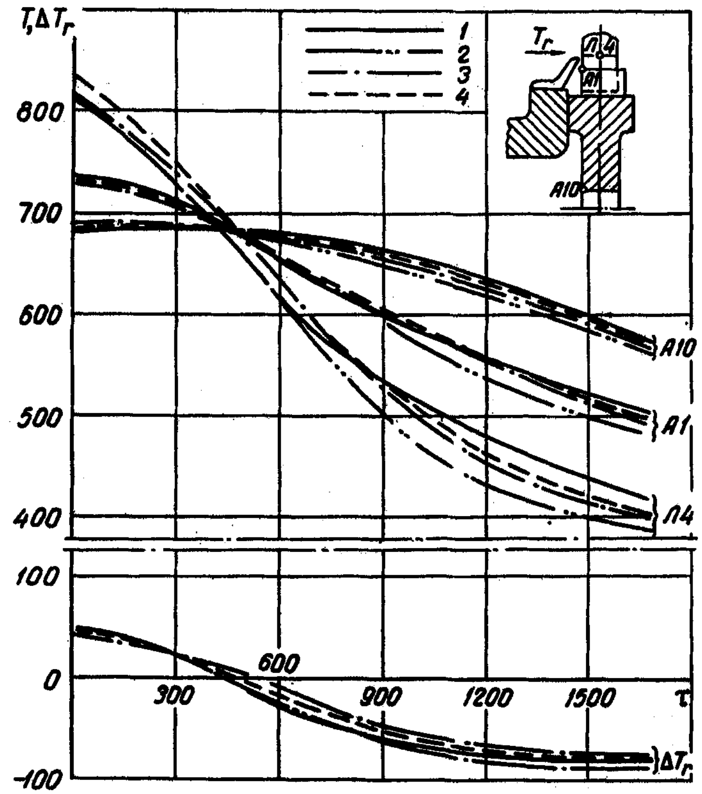

Fig. 2. Results of calculated cooling of the rotor of a gas turbine: 1) according to the Liebmann method; 2, 3, 4) respectively, the 1st, 2nd, and 3rd approximations according to the method of this article; T in ° K, τ in sec.

2) The system of boundary conditions (T_m and the boundary resistances) is drawn up, the values of the boundary resistances being calculated according to the corresponding α values of the initial conditions, while the temperature of the medium is calculated according to (7).

3) Measurements are made.

4) Preparation of solution of the next step reduces to conversion and assignment of new values of the medium temperature T_m^1.

The approximation method, which allows us in solving an unsteady problem by the Liebmann method to take into account variation of the heat transfer coefficients without variation of the boundary resistances which simulate the convective heat transfer, was employed and gave good results in solving methodological problems in the cooling of an infinite cylinder, whose conditions have already been described above. The largest deviation from the results of calculation by the Liebmann method with α = var do not exceed 3.5% of the maximum temperature level in the cylinder.

The method of successive approximations for solution of unsteady heat conduction problems on RC networks has been used to solve more complex problems. The unsteady temperature fields were obtained for the rotor of a natural gas turbine in a check condition, the boundary and initial conditions being calculated according to data from one of the tests carried out on this machine. The results of the calculation were verified from the readings of 69 thermocouples in the calculation time intervals. Figure 2 shows the variation of temperature according to the test results, for typical points on the rotor (A1, A10, L4), as well as giving the results according to each calculation approximation performed. As early as the third approximation, the maximum deviation between the test and the calculated temperatures did not exceed 1.5% of the maximum temperature level in the rotor.

The investigation performed indicates that the method of successive approximations described may be recommended for application on existing equipment with RC networks, for calculating the variation of heat transfer coefficients, in solution of unsteady heat conduction problems.

In solving problems by the Liebmann method on complex models with developed boundary conditions of the third kind, an approximation method of calculating the variation of heat transfer coefficients may be useful, since the difficulty of solution then decreases, and the possibility of errors in reassignment of boundary resistances is excluded.

NOTATION

α_0 and α_i are the heat transfer coefficients corresponding to the initial conditions and the i-th time interval; T_{mi} is the temperature of medium for i-th time interval; T_{0i} is the temperature of surface for i-th time interval obtained with $\alpha = \alpha_0$ = const; T_i is the temperature of surface for i-th time interval obtained with α = var; T_{i-1} is the temperature at one of points lying on the boundary of the object.

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